Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Show that the equation $x = 7 + 2x^2$ has no real roots. **[3]**

① $x = 7 + 2x^2$

$0 = 2x^2 - x + 7$

② $a = 2, \; b = -1, \; c = 7$

$\therefore$ discriminant $= b^2 - 4ac$

$= (-1)^2 - 4(2)(7)$

$= 1 - 56$

$= -55$

③ $-55 < 0 \; \therefore \; x = 7 + 2x^2$ has no real roots

DISCRIMINANT $= b^2 - 4ac$

if $b^2 - 4ac < 0$ then no real roots.

2 In this question you must show detailed reasoning.

Fig. 2 shows the graphs of $y = 4\sin x°$ and $y = 3\cos x°$ for $0 \leqslant x \leqslant 360$.

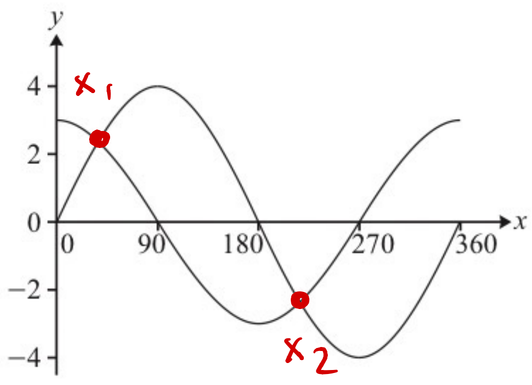


Fig. 2

Find the x-coordinates of the two points of intersection, giving your answers correct to 1 decimal place. **[3]**

Eq ① $y = 4\sin x$

Eq ② $y = 3\cos x$

① Set the two equations equal to each other

$$4\sin x = 3\cos x$$

② rearrange and simplify

$$\frac{\sin x}{\cos x} = \frac{3}{4} \rightarrow \tan x = \frac{3}{4}$$

③ Solve $\tan x = 3/4$

$$x = \tan^{-1}(3/4)$$

$$x_1 = 36.86989\ldots$$

$$x_2 = 180 + x_1 = 216.86989\ldots$$

MAKE SURE YOU'RE IN DEGREES!

④ Check these are valid with Fig. 2

$$\therefore \; x_1 = 36.9°, \; x_2 = 216.9°$$

3 Given that k is an integer, express $\dfrac{3\sqrt{2}-k}{\sqrt{8}+1}$ in the form $a+b\sqrt{2}$ where a and b are rational expressions in terms of k. **[4]**

① Rewrite the surd $\sqrt{8}$ so it's in the form $x\sqrt{2}$

$$\sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$$

$$\rightarrow \quad \frac{3\sqrt{2}-k}{2\sqrt{2}+1}$$

② Rationalise the fraction by multiplying it with the conjugate.

$$\frac{(3\sqrt{2}-k)(2\sqrt{2}-1)}{(2\sqrt{2}+1)(2\sqrt{2}-1)} = \frac{12-3\sqrt{2}-2k\sqrt{2}+k}{8-\cancel{2\sqrt{2}+2\sqrt{2}}-1}$$

↳ these cancel

③ Simplify so in the form $a+b\sqrt{2}$

$$\frac{12-3\sqrt{2}-2k\sqrt{2}+k}{7} = \frac{12+k}{7} - \frac{3\sqrt{2}+2k\sqrt{2}}{7}$$

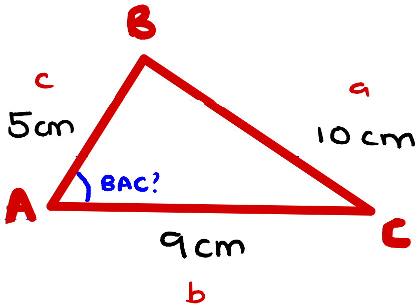
$$\therefore = \frac{12+k}{7} - \frac{3+2k}{7}\sqrt{2}$$

where $\dfrac{12+k}{7} = a$ and $-\dfrac{3+2k}{7} = b$

4 A triangle ABC has sides AB = 5 cm, AC = 9 cm and BC = 10 cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms. [2]

(b) Find the exact area of the triangle. [3]



a) ① write down the cosine rule

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

2. ② substitute in our values

$$\cos(BAC) = \frac{9^2 + 5^2 - 10^2}{2(9)(5)}$$

$$= \frac{81 + 25 - 100}{90} = \frac{6}{90}$$

PRO TIP: change equation for your question → in our case ∠BAC = ∠A so no need

③ we need to find cos (BAC), so just simplify 6/90

$\cos(BAC) = 6/90$

$\therefore \cos(BAC) = 1/15$

b) ① write down formula for the area of a triangle

$\text{area} = 1/2\, ab \sin C \rightarrow \frac{1}{2} bc \sin A$

② Find Sin C

$\sin BAC = \sqrt{1 - \cos BAC^2} = \sqrt{1 - (1/15)^2} = \frac{4\sqrt{14}}{15} = \sin A$

③ Find area using the formula

$\text{area} = \frac{1}{2} \times 5 \times 9 \times \frac{4\sqrt{14}}{15}$

$= 6\sqrt{14}$ cm

5 In this question, the unit vectors **i** and **j** are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

j ↑ → i

(a) Write the weight of the particle as a vector. **[1]**

The particle moves under the action of its weight and two external forces $(3\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} + 18\mathbf{j})$ N.

(b) Find the acceleration of the particle, giving your answer in vector form. **[2]**

a) $W = mg$

$W = 2.5 \times 9.8 = 24.5$

weight acts downwards only so is $-\mathbf{j}$

$\therefore \quad W = -24.5\mathbf{j}$ N

b) N2L : $F = ma$

$$\begin{pmatrix} 0 \\ -24.5 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 18 \end{pmatrix} = 2.5a$$

$$\begin{pmatrix} 2 \\ -8.5 \end{pmatrix} = 2.5a$$

$$\therefore \quad a = \begin{pmatrix} 0.8 \\ -3.4 \end{pmatrix} = 0.8\mathbf{i} - 3.4\mathbf{j}$$

6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

1 ton = 1000 kg

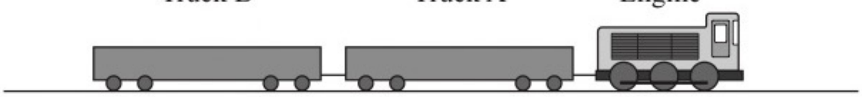


Fig. 6

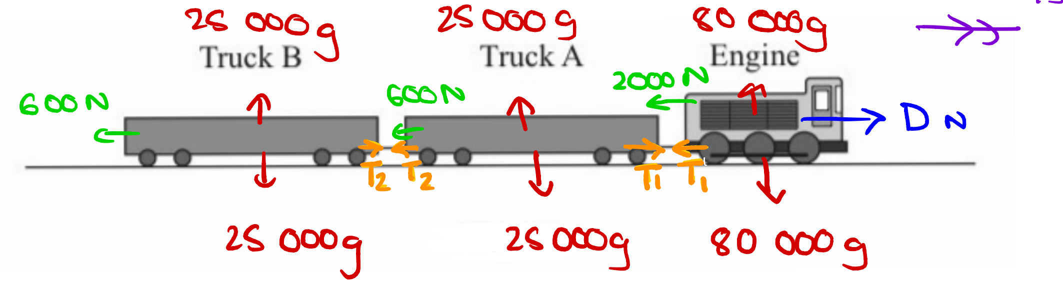
The engine exerts a driving force of D N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of $0.1\,\mathrm{m\,s^{-2}}$.

(a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. **[3]**

(b) Calculate the value of D. **[2]**

(c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. **[2]**

a)



b) Write an equation using N2L for the train

$$\underbrace{D - 2000 - \cancel{T_1}}_{\text{Engine}} + \underbrace{\cancel{T_1} - 600 - \cancel{T_2}}_{\text{Truck A}} + \underbrace{\cancel{T_2} - 600}_{\text{Truck B}} = ((25000 \times 2) + 80000)(0.1)$$

$$D - 2000 - 600 - 600 = 130000(0.1)$$

$$D - 3200 = 13000$$

$$D = 16200\text{ N}$$

c) ① Engine + Truck A.

$$16200 - 2000 - T_1 = 80000(0.1)$$

$$14200 - T_1 = 8000 \rightarrow T_1 = 6200\text{ N}$$

② Truck A + Truck B

$$T_2 - 600 = 25000(0.1)$$

$$T_2 - 600 = 2500 \rightarrow T_2 = 3100\text{ N}$$

T_1 is 3100 N bigger than T_2

7 **In this question you must show detailed reasoning.**

(a) Nigel is asked to determine whether $(x+7)$ is a factor of $x^3-37x+84$. He substitutes $x=7$ and calculates $7^3-37\times7+84$. This comes to 168, so Nigel concludes that $(x+7)$ is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that $(x+7)$ is a factor of $x^3-37x+84$. **[2]**

(b) Sketch the graph of $y=x^3-37x+84$, indicating the coordinates of the points at which the curve crosses the coordinate axes. **[5]**

(c) The graph in part **(b)** is translated by $\begin{pmatrix}1\\0\end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y=x^3+ax^2+bx+c$ where a, b and c are integers. **[4]**

a) ① $x+7=0$ means we should substitute in $x=-7$ not $x=7$ therefore the rest of his conclusion can not be correct as it shows $(x-7)$ is not a factor

② Showing that $(x+7)$ is a factor

(Substitute -7) $(-7)^3-(37\times-7)+84$

$= -343-(-259)+84$

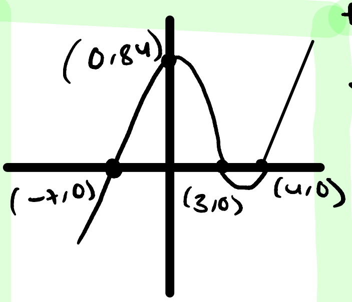
$= -343+343$

$= 0$ $\therefore$ $(x+7)$ is a factor of $x^3-37x+84$

b) ① we know that $(x+7)$ is a factor

$+84$ is the y-intercept $\rightarrow$ $(0,84)$

x^3 $\rightarrow$ so we know it's cubic



	x^2	$-7x$	$+12$
x	x^3	$-7x^2$	$+12x$
$+7$	$7x^2$	$-49x$	84

$\rightarrow$ $(x+7)(x^2-7x+12)$

$(x+7)(x-4)(x-3)$

$\therefore$ $(-7,0),(+4,0)(+3,0)$ are the other roots

c) ① Translated by $\begin{pmatrix}1\\0\end{pmatrix}$ means:

means that (x) is now $(x-1)$

② substitute $(x-1)$

$(x-1)^3-37(x-1)+84$

$=(x-1)(x-1)(x-1)-37x+37+84$

$=(x^2-2x+1)(x-1)-37x+121$

$=(x^3-3x^2+3x-1)-37x+121$

$=x^3-3x^2-34x+120$

8 In this question you must show detailed reasoning.

Show that the only stationary point on the graph of $y = x^2 - 4\sqrt{x}$ is a minimum point at $(1, -3)$.

[7]

$y = x^2 - 4\sqrt{x}$

① Rewrite the equation

$y = x^2 - 4x^{1/2}$

② Find dy/dx

$\frac{dy}{dx} = 2x - 2x^{-1/2}$

③ Solve $\frac{dy}{dx} = 0$

$2x - 2x^{-1/2} = 0 \rightarrow 2x = 2x^{-1/2}$

$\rightarrow 2x^1 = \frac{2}{x^{1/2}}$

$2x^1 x^{1/2} = 2$

$x^{3/2} = 1$

$\therefore x = 1 \rightarrow$ as there is only one solution there can only be one stationary point.

④ Find y when $x = 1$

$y = 1^2 - 4\sqrt{1}$

$y = 1 - 4$

$y = -3$ $\therefore (1, -3)$ is the stationary point

⑤ Find if $(1, -3)$ is a minimum point

$\frac{d^2y}{dx^2} = 2 + x^{-3/2}$

when $x = 1$, $\frac{d^2y}{dx^2} = 2 + 1^{-3/2} = 3$

$3 > 0$ $\therefore$ it is a minimum point

9 In this question you must show detailed reasoning.

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is 25.6 m s^{-1}.

In one model for the motion, the velocity v m s^{-1} at time t seconds is given by $v = 1.2t^2 - kt^3$, where k is a constant and $0 \leqslant t \leqslant 8$.

(a) The model gives the correct velocity of 25.6 m s^{-1} at time 8 s. Show that $k = 0.1$. **[2]**

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of 25.6 m s^{-1} at time 8 s. **[2]**

(c) Show that these two models give the same value for the displacement in the first 8 s. **[5]**

a) ① substitute 25.6m/s and 8s into the model

$v = 1.2t^2 - kt^3$

$25.6 = 1.2(8^2) - k(8^3)$

$25.6 = 76.8 - 512k$ (rearrange)

$512k = 51.2$ (solve)

$k = \frac{51.2}{512} = 0.1$ as required

b) constant acceleration = SUVAT formulae

we have t, v, u and need to find a

$\therefore$ $v = u + at$

(substitute values in) $\rightarrow$ $25.6 = 0 + 8a$

$a = \frac{25.6}{8} = 3.2$ m/s

c) ① $\int_{t_2}^{t_2} v = s$ for model one.

$\int_0^8 1.2t^2 - \frac{1}{10}t^3 \, dt$

$= \left[0.4t^3 - 0.025t^4\right]_0^8$

$= \left[(0.4(8^3) - 0.025(8^4)\right] - \left[0.4(0^3) - 0.025(0^4)\right]$

$= [204.8 - 102.4] - 0 = 102.4$ m

② SUVAT $s = \frac{1}{2}(u+v)t$ for model two

$s = \frac{1}{2}(0+25.6)8 = 4(25.6) = 102.4$m

$\therefore$ both model one and model two have the same displacement

10 In this question you must show detailed reasoning.

(a) Sketch the gradient function for the curve $y = 24x - 3x^2 - x^3$. **[5]**

(b) Determine the set of values of x for which $24x - 3x^2 - x^3$ is decreasing. **[2]**

a) ① find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 24 - 6x - 3x^2$$

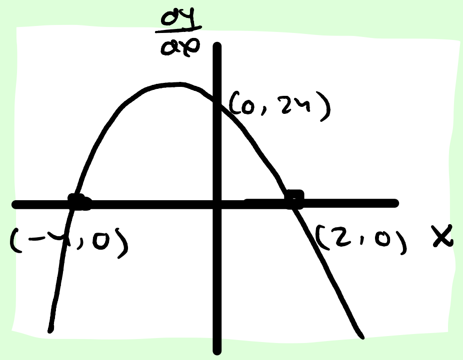
② Solve $\frac{dy}{dx} = 0$

$24 - 6x - 3x^2 = 0 \rightarrow 3x^2 + 6x - 24 = 0$

$\rightarrow x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$\therefore x = -4, \; x = 2$



b) For $24x - 3x^2 - x^3$ to be decreasing $\frac{dy}{dx} < 0$

$3x^2 + 6x - 24 < 0$

$(x+4)(x-2) < 0$

$x < -4$ OR $x > 2$

11 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. **[3]**

(b) Explain why this model

(i) is not suitable for small values of t, **[1]**

(ii) cannot be used to find the time for the block to melt completely. **[1]**

David instead proposes a linear model $m = at + b$, where a and b are constants.

(c) Find the values of the constants for which the model fits the mass of the block when $t = 5$ and $t = 50$. **[3]**

(d) Interpret these values of a and b. **[2]**

(e) Find the time according to this model for the block of ice to melt completely. **[1]**

END OF QUESTION PAPER

a) $m = \frac{k}{t}$

① find k, using $t = 5$

$2.1 = \frac{k}{5} \rightarrow k = 10.5$

② verify using $t = 50$

$m = \frac{10.5}{50} = 0.21$ ∴ this model fits his measurements

b) i. when t is small $\frac{10.5}{t}$ becomes large so mass will not be modelled correctly.

ii. melted completely means $m = 0$ but $\frac{10.5}{0}$ is undefined so the model can't be used

c) ① $2.1 = 5a + b$

② $0.21 = 50a + b$

② − ① $0.21 - 2.1 = 50a - 5a + \cancel{b - b}$

$-1.89 = 45a$

$a = -0.042$

⇒ ① $2.1 = 5(-0.042) + b$

$2.1 = -0.21 + b$

$b = 2.31$

checking:

$0.21 = 50(-0.042) + 2.31$

$0.21 = 0.21$

so $a = -0.042$ and $b = 2.31$

a) a: a is the rate at which ice melts
0.042 kg of ice is lost per hour

b: b is the initial mass of the block
which is 2.31 kg

c) $m = -0.042t + 2.31$

$0 = -0.042t + 2.31$

$0.042t = 2.31$

$t = 55$ hours

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